Supplementary file

Multi-factorial predictive model linking acoustic characteristics with geotechnical parameters in deep-water shallow formations

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Appendix A: Biot-Stoll theory

Gassmann (1951) established a theoretical model for the propagation of elastic waves in porous media based on the theory of pore consolidation. Biot (1955; 1962; 1968) proposed the theory of elastic wave propagation in water-saturated soils and established the theory of elastic wave dynamics in water-saturated pore media, which laid the foundation for the study of the geotechnical properties of sediments using acoustic waves. The fluctuation equation of the Biot theory is expressed as:

$$\nabla^2(H\varepsilon - C) = \frac{\partial^2}{\partial t^2}(\rho\varepsilon - \rho_f\zeta) \tag{1}$$

$$\nabla^2(C\varepsilon - M\zeta) = \frac{\partial^2}{\partial t^2}(\rho_f - m\zeta) - \frac{F\eta}{\kappa}\frac{\partial\zeta}{\partial t} \tag{2}$$

where ε is the volume strain of the soil frame, ζ is the volume increment of the fluid entering or leaving the frame, t is the propagation time of the P-wave, ρ is the density of the soil frame, ρ_f is the density of the fluid, η is the viscosity coefficient, and κ is the permeability.

The expressions for the Biot parameters were developed based on measurable sediment properties. The Biot theoretical modulus of elasticity H, additional modulus of elasticity C, and complex elastic modulus M were calculated using the porosity ϕ, frame bulk modulus K_b, frame shear modulus μ_b, modulus of pore fluid K_f , and grain bulk modulus Kr, which is expressed as follows:

$$\begin{gathered} \mathrm{D} = K_r[1 + \phi(K_r/K_f - 1)] \\ \mathrm{M} = K_r^2(D - K_b)^{-1} \\ \mathrm{C} = K_r(K_r - K_b)(D - K_b)^{-1} \\ \mathrm{H} = K_b + \frac{4}{3}\mu_b + (K_r - K_b)^2(D - K_b)^{-1} \end{gathered} \tag{3}$$

where K_f and K_r are the real numbers and K_b and μ_b are complex numbers. K_b = $K_{b0}(1+i\delta_k)$ and μ_b = $\mu_{b0}(1+i\delta_\mu)$. δ_k are the dissipation coefficient of the bulk modulus and δ_μ is the dissipation coefficient of the shear modulus.

Stoll (1989) applied the Biot theory to calculate the acoustic velocity and attenuation in submarine sediment media. The traveling equation of a harmonic plane wave through a porous medium is expressed as:

$$\begin{vmatrix} Hk^2 - \rho\omega^2 & \rho_w\omega^2 - Ck^2 \\ Ck^2 - \rho_w\omega^2 & m\omega^2 - Mk^2 - i\dfrac{\omega F\eta}{\kappa} \end{vmatrix} = 0 \tag{4}$$

where ρ is the density of the shallow sediments in deep-water, ρ_w is the density of the water in the pores, k is the complex wavenumber, η is the viscosity coefficient, κ is the permeability, ω is the angular frequency, m is the effective density of fluid in the pores, $m = a'\rho\omega/\phi$, a' is the tortuosity, i is the imaginary unit, and F is the correction factor of viscosity related to the frequency.

The wavenumbers of the fast and slow waves are obtained by solving k as follows:

$$(Hk^2 - \rho\omega^2)\left(m\omega^2 - Mk^2 - i\frac{\omega F\eta}{\kappa}\right) - (\rho_w\omega^2 - Ck^2)(Ck^2 - \rho_w\omega^2) = 0 \tag{5}$$

The above equation is transformed into:

$$(-HM + C^2)k^4 - \rho m\omega^4 + j\frac{\rho F\eta\omega^3}{\kappa} + \rho_w^4 + \left(Hm\omega^2 + \rho\omega^2 M - jH\frac{\omega F\eta}{\kappa} - 2C\rho_w^2\right)k^2 = 0 \tag{6}$$

The above expression is a quadratic expression of k^2:

$$\begin{gathered} a(k^2)^2 + bk^2 + c = 0 \\ a =- HM + C^2 \\ b = Hm\omega^2 + \rho\omega^2 M - jH\frac{\omega F\eta}{\kappa} - 2C\rho_w^2 \\ c =- \rho m\omega^4 + j\frac{\rho F\eta\omega^3}{\kappa} + \rho_w^4 \end{gathered} \tag{7}$$

Thus, the solution of the complex wave number is:

$$k^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{8}$$

where + denotes the complex wave number solution of the fast wave and − denotes the complex wave number solution of the slow wave. The relationship between the complex wavenumber, velocity, and attenuation efficiency is expressed as:

$$k = \frac{\omega}{v} + j\alpha \tag{9}$$

Thus, the acoustic velocity v and attenuation efficient α are calculated as follows:

$$v = \frac{\omega}{Re(k)} \tag{10}$$

$$\alpha = Im(k) \tag{11}$$

where $Re(k)$ is the real part of the complex number k and $Im(k)$ is its imaginary part.

Appendix B: Mathematical relationship between porosity and clay content

According to the water content calculation formula, the expressions for the total mass of the sediment, the mass of sand particles, and the mass of clay particles can be derived as follows:

$$m = \frac{m_w(1+\varpi)}{\varpi} \tag{12}$$

$$m_s = \frac{m_w(1-c)}{\varpi} \tag{13}$$

$$m_c = \frac{m_w c}{\varpi} \tag{14}$$

where m is the weight of the sediments, m_w is the weight of water, ϖ is the water content, m_s is the weight of sand particles, c is the clay content, m_c is the weight of clay particles.

The mathematical expressions for the volume of each constituent are provided below:

$$V_w = \frac{m}{\rho}\emptyset = \frac{m_w(1+\varpi)\emptyset}{\rho\varpi} \tag{15}$$

$$V_s = \frac{m_s}{\rho_s} = \frac{m_w(1-c)}{\varpi\rho_s} \tag{16}$$

$$V_c = \frac{m_c}{\rho_c} = \frac{m_w c}{\varpi\rho_c} \tag{17}$$

where V_w is the volume of water, ρ is the density of shallow sediments, ϕ is the porosity, ρ_s is the density of sand particle, ρ_c is the density of clay particle, V_s is the volume of sand and V_c is the volume of clay particles.

The above formulas are substituted into the density formula:

$$\rho = \frac{m}{V_w+V_s+V_c} \tag{18}$$

Under conditions of constant water content, the mathematical relationship between porosity and clay content can be articulated as follows:

$$\phi = 1 - \frac{\rho(1-c)}{\rho_s(1+\varpi)} - \frac{\rho c}{\rho_c(1+\varpi)} \tag{19}$$

Appendix C: Process of Wavelet analysis

Wavelet analysis, a well-established method in signal processing (Mallat, 1999), was employed in this study to discern the precise arrival time of the initial acoustic wave. Wavelet analysis is instrumental in decomposing the original acoustic signal through the utilization of wavelet transform coefficients. Following the arrival of the acoustic signal at the receiver transducer, the detailed signal undergoes a wavelet transformation, ultimately yielding the modal maximum line. This modal maximum line aids in pinpointing the point of abrupt change or mutation within the original acoustic signal, which corresponds to the arrival time of the acoustic signal. The process of wavelet transformation can be mathematically expressed as follows:

$$WT(u,z) = \int_{-\infty}^{\infty} f(t)\overline{\psi(t)}\,dt = \frac{1}{\sqrt{z}}\int_{-\infty}^{\infty} f(t)\overline{\psi\left(\frac{t-u}{z}\right)}dt \tag{20}$$

$$\psi(t) = \frac{1}{\sqrt{z}}\psi\left(\frac{t-u}{z}\right) \tag{21}$$

where $\psi(t)$ is the generating function of the wavelet transformation, u is the translation parameter of the wavelet transformation, $\psi(t)$ is the conjugate function of $\psi(t)$ and z is the scale parameter of the wavelet transformation.

After the arrival time of the first wave t is read through the wavelet transformation and the distance of the acoustic wave penetrating the simulated formation is noted, the sound wave velocity is calculated using the following formula:

$$V_p = \frac{h}{t} \tag{22}$$

where h is the distance between the two transducers and t is the propagation time of the acoustic wave in a saturated medium.

As the pore water in the simulated formation was sensitive to changes in temperature and pressure, the measured data changed during the measurement of the acoustic velocity in the laboratory. Therefore, in this study, it was necessary to correct the measured acoustic velocity in the laboratory and restore it to the acoustic velocity value of the deep-water shallow formation as much as possible. The acoustic velocity ratio, R, is defined as the ratio of the acoustic velocity of porous media to that of water under the same temperature and pressure conditions. This value is considered to remain constant in general. The measured results of the acoustic velocity of the simulated formation were corrected according to the acoustic velocity of seawater under specific temperature and pressure conditions. The formula for the acoustic velocity ratio is as follows:

$$R = \frac{V_p}{V_w} = \frac{V_{pi}}{V_{wi}} \tag{23}$$

where V_{pi} is the acoustic velocity of the formation under the conditions of the in situ temperature and pressure, and V_{wi} is the acoustic velocity of seawater under the in situ temperature and pressure conditions.

Appendix D: Introduction of Single-factor variance (ANOVA)

ANOVA is a method proposed by Fisher (1992) to test the significance of differences between two or more samples. Its basic idea is to divide the total variation of the measured data into experiment condition (inter-group) effect and error (intra-group) effect according to the source of variation, and make their quantity estimation, so as to determine the influence of experimental treatment on the research results. In this study, the one-way ANOVA technique was selected to investigate the impact of individual factors on acoustic characteristics. The specific procedural steps for one-way ANOVA are delineated as follows.

- Step 1 Make assumptions.

The underlying assumptions for this analysis encompass the presumption that each set of measurement data conforms to a normal distribution, that the variances within each data group are equivalent, and that all experimental samples are independent and mutually exclusive.

- Step 2 Select the test statistic.

The summation of squared deviations (ST) is a frequently employed metric for quantifying the dispersion within a dataset. In the context of a given experiment, this summation encapsulates the magnitude of variation among all data points, expressed as the sum of squares of population deviations:

$$ST = \sum_{i=1}^{i=r} \sum_{j=1}^{j=m} (y_{ij} - \bar{y})^2 \tag{24}$$

The degree of freedom is $dF_T = n - 1$, n is the number of all measurements, $n = r * m$, $\bar{y}$ is the average value of all measured data.

Employ the summation of squared intra-group deviations, commonly referred to as the summation of squared error deviations, to delineate the disparity inherent to each level (or group) stemming solely from random error:

$$Se = \sum_{i=1}^{i=r} \sum_{j=1}^{j=m} (y_{ij} - \bar{y}_i)^2 \tag{25}$$

where, y_i is the mean value of Group i measurements, the degree of freedom is $dF_e = r(m - 1) = n - r$.

In tandem with the random errors manifesting between groups, there also exist distinctions attributed to various effects across distinct levels of factors (or groups). These distinctions can be quantified through the summation of squared deviations between groups, commonly referred to as the summation of squared deviations of an independent variable:

$$SA = m \sum_{i=1}^{i=r} (\bar{y}_i - \bar{y})^2 \tag{26}$$

the degree of freedom is $dF_A = r - 1$.

The expression of the relationship between the above sum of squares of the deviations is as follows:

$$ST = Se + SA \tag{27}$$

the degree of freedom is $dF_t = dF_e + dF_A$.

The formulation of the mean square serves to mitigate the influence arising from disparate degrees of freedom, thereby facilitating the comparative analysis of the summation of squared deviations among sample groups. Consequently, the test statistic denoted as "F" can be articulated as the ratio between the mean square of the summation of squared deviations between groups and the mean square of the summation of squared deviations within groups:

$$F = \frac{MS_A}{MS_e} = \frac{S_A/df_A}{S_e/df_e} \tag{28}$$

- Step 3 Give the rejection domain and make a judgment.

The value of the test statistic F established by Eq. (28) follows the F distribution with degrees of freedom df_A and df_e. The larger the value of F is when the more inclined to reject the null hypothesis, so the rejection field W of this test is:

$$W = F \geqslant F_{1-\chi}(df_A, df_e) \tag{29}$$

where, χ is the significant level, which is generally 0.05. $F_{1-\chi}$ (df_A, df_e) can be obtained by the table of F distribution. If $F \geqslant F_{1-\chi}$ (df_A, df_e), it means that there is a significant difference between the groups of factors. If $F \leqslant F_{1-\chi}$ (df_A, df_e), the difference between the factor groups is not significant.

The P-value of this test can be obtained from the density function corresponding to the F distribution:

$$P = P(Y \geqslant F) \tag{30}$$

In practice, if the value of P is small (when $P \leqslant 0.001$), it substantiates the basis for rejecting the null hypothesis, indicating a substantial influence of the parameter on the dependent variable. In contrast, if $P \geqslant 0.05$, it is generally construed that the said parameter exerts a negligible effect on the dependent variable.

Appendix E: Fitting results between geo-parameters and acoustic characteristics

Table **S1**. Corresponding fitting effects of the relationship between clay content and acoustic characteristics.

Form of function	Expression of function	DFE	SSE	RMSE	R^2
Polynomial	$v = 1344 - 16.96c$	59	0.0729	0.0352	0.9999
	$\alpha = 0.89 - 0.19c$	59	0.3164	0.0732	0.8260
Exponential	$v = 1344\mathrm{e}^{-0.013c}$	60	0.0923	0.0392	0.9998
	$\alpha = 0.02\mathrm{e}^{4.39c}$	59	0.0308	0.0229	0.9831
Power	$v = 1349 - 0.72c^{0.61}$	59	2.0319	0.1856	0.9963
	$\alpha = 1.73c^{4.63} + 0.099$	58	0.0083	0.0120	0.9954
Logarithmic	$v = 1330 - 7.55\ln c$	59	15.109	0.5060	0.9722
	$\alpha = 0.37\ln c + 0.53$	59	0.5548	0.6898	0.6950

Table **S2**. Corresponding fitting effects of the relationship between density and acoustic characteristics.

Form of function	Expression of function	DFE	SSE	RMSE	R^2
Polynomial	$v = 400\rho^2 - 1279\rho + 2247$	129	928.08	2.692	0.9995
	$\alpha = 17.31\rho^2 - 80.21\rho + 92.28$	129	0.2350	0.0588	0.9998
Exponential	$v = 1011\mathrm{e}^{-0.2\rho}$	129	3.7×10^5	53.67	0.7903
	$\alpha = 620.9\mathrm{e}^{-2.5\rho}$	129	638.20	2.2243	0.9658
Power	$v = 1325 + 1.12\rho^{6.8}$	129	2.6×10^4	14.13	0.9856
	$\alpha = 74.1\rho^{-1.4} - 24.7$	129	213.20	1.2907	0.9886
Logarithmic	$v = 1198 - 427\ln\rho$	129	5.8×10^5	66.91	0.6741
	$\alpha = -52\ln\rho + 40.5$	129	627.60	2.2057	0.9664

Table **S3**. Corresponding fitting effects of the relationship between water content and acoustic characteristics.

Form of function	Expression of function	DFE	SSE	RMSE	R^2
Polynomial	$v = 13.42\varpi^2 - 18.74\varpi + 1329$	32	2.6961	0.290	0.9269
	$\alpha = -2.79\varpi^2 + 3.9\varpi - 0.28$	32	0.1122	0.0592	0.9303
Exponential	$v = 32.14\mathrm{e}^{-11\varpi} + 1323$	31	0.0221	0.0267	0.9994
	$\alpha = 0.9\mathrm{e}^{-0.2\varpi} - 6.28\mathrm{e}^{-10.6\varpi}$	31	0.0009	0.0054	0.9994
Power	$v = 1322 + 0.16\varpi^{-2.074}$	32	0.0001	0.0001	0.9999
	$\alpha = -0.035\varpi^{-2.027} + 1.116$	32	0.0001	0.0001	0.9999
Logarithmic	$v = 1321 - 2.25\ln\varpi$	33	5.5741	0.4110	0.8488
	$\alpha = 0.4714\ln\varpi + 1.229$	33	0.2350	0.0844	0.8541

Table **S4**. Corresponding fitting effects of the relationship between shear strength and acoustic characteristics.

Form of function	Expression of function	DFE	SSE	RMSE	R^2
Polynomial	$v = 0.0032s^2 + 0.153s + 1331$	26	6.4172	0.4968	0.9997
	$\alpha = 0.0011s^2 - 0.4909s + 50.91$	26	0.0662	0.0505	0.9999
Exponential	$v = 32.48e^{-0.0093s} + 1296$	26	30.7141	1.0869	0.9987
	$\alpha = -57.88e^{-0.0153s}$	27	135.7	2.242	0.9743
Power	$v = 1333 + 0.0156s^{1.74}$	26	1.6565	0.2524	0.9999
	$\alpha = -2.7414s^{0.61} + 58.27$	26	5.537	0.4615	0.9990
Logarithmic	$v = 34.83 \ln s + 1224$	27	5359	14.09	0.7673
	$\alpha = -18.56 \ln s + 98$	27	261.4	3.112	0.9505

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